Preface

In this thesis we study a variety of problems in superfluid turbulence, principally in two dimensions. A summary of the main results of our studies is given below; we indicate the Chapters in which we present these.

In Chapter 1, we provide an overview of several problems in superfluid turbulence with special emphasis on background material for the problems we study in this thesis. In particular, we give: (a) a brief introduction of fluid turbulence; (b) an overview of superfluidity and the phenomenological two-fluid model; (c) a brief overview of experiments on superfluid turbuelnce; (d) an introductory accounts of the phenomenological models used in the study of superfluid turbuelnce. We end with a summary of the problems we study in subsequent Chapters of this thesis.

In Chapter 2, we present a systematic, direct numerical simulation of the two-dimensional, Fourier-truncated, Gross-Pitaevskii equation to study the turbulent evolutions of its solutions for a variety of initial conditions and a wide range of parameters. We find that the time evolution of this system can be classified into four regimes with qualitatively different statistical properties. First, there are transients that depend on the initial conditions. In the second regime, power- law scaling regions, in the energy and the occupation-number spectra, appear and start to develop; the exponents of these power laws and the extents of the scaling regions change with time and depend on the initial condition. In the third regime, the spectra drop rapidly for modes with wave numbers $k > k_c$ and partial thermalization takes place for modes with $k < k_c$; the self-truncation wave number $k_c(t)$ depends on the initial conditions and it grows either as a power of t or as $\log t$. Finally, in the fourth regime, complete thermalization is achieved and, if we account for finite-size effects carefully, correlation functions and spectra are consistent with their nontrivial Berezinskii-Kosterlitz-Thouless forms. Our work is a natural generalization of recent studies of thermalization in the Euler and other hydrodynamical equations; it combines ideas from fluid dynamics and turbulence, on the one hand, and equilibrium and nonequilibrium statistical mechanics on the other.

In Chapter 3, we present the first calculation of the mutual-friction coefficients α and α' (which are parameters in the Hall-Vinen-Bekharevich-Khalatnikov two-fluid model that we study in chapter 5) as a function of temperature in a homogeneous Bose gas in two-dimensions by using the Galerkin-truncated Gross-Pitaevskii equation, with very special initial conditions, which we obtain by using the advective, real, Ginzburg-Landau equation (ARGLE) and an equilibration procedure that uses a stochastic Ginzburg-Landau equation (SGLE). We also calculate the normal-fluid density as a function of temperature.

In Chapter 4, we elucidate the interplay of particles and fields in superfluids, in both simple and turbulent flows. We carry out extensive direct numerical simulations (DNSs) of this interplay for the two-dimensional (2D) Gross-Pitaevskii (GP) equation. We obtain the following results: (1) the motion of a particle can be chaotic even if the superfluid shows no sign of turbulence; (2) vortex motion depends sensitively on particle charateristics; (3) there is an effective, superfluid-mediated, attractive interaction between particles; (4) we introduce a short-range repulsion between particles, with range $r_{\rm SR}$, and study two- and many-particle collisions; in the case of two-particle, head-on collisions, we find that, at low values of $r_{\rm SR}$, the particle collisions are inelastic with coefficient of restitution $e = 0$; and, as we increase $r_{\rm SR}$, e becomes nonzero at a critical point, and finally attains values close to 1; (5) assemblies of particles and vortices show rich, turbulent, spatio-temporal evolution.

In Chapter 5, we present results from our direct numerical simulations (DNSs) of the Hall-Vinen-Bekharevich-Khalatnikov (HVBK) two-fluid model in two dimensions. We have designed these DNSs to study the statistical properties of inverse and forward cascades in the HVBK model. We obtain several interesting results that have not been anticipated hitherto: (1) Both normal-fluid and superfluid energy spectra, $E^n(k)$ and $E^s(k)$, respectively, show inverse- and forward-cascade regimes; the former is characterized by a power law $E^s(k) \sim E^n(k) \sim k^{-\alpha}$ whose exponent is consistent with $\alpha \simeq 5/3$. (2) The forward-cascade power law depends on (a) the friction coefficient, as in 2D fluid turbulence, and, in addition, on (b) the coefficient B of mutual friction, which couples normal and superfluid components. (3) As B increases, the normal and superfluid velocities, $\mathbf{u}_n$ and $\mathbf{u}_s$, respectively, get locked to each other, and, therefore, $E^s(k) \simeq E^n(k)$, especially in the inverse-cascade regime. (4) We quantify this locking tendency by calculating the probability distribution functions (PDFs) $\mathcal{P}(\cos(\theta))$ and $\mathcal{P}(\gamma)$, where the angle $\theta \equiv (\mathbf{u}_{\rm n} \cdot \mathbf{u}_{\rm s})/(|\mathbf{u}_{\rm n}||\mathbf{u}_{\rm s}|)$ and the amplitude ratio $\gamma = |\mathbf{u}_{\rm n}|/|\mathbf{u}_{\rm s}|$; the former has a peak at $\cos(\theta) = 1$; and the latter exhibits a peak at $\gamma = 1$ and power-law tails on both sides of this peak. (4) This locking increases as we increase B, but the power-law exponents for the tails of $\mathcal{P}(\gamma)$ are universal, in so far as they do not depend on B, ρ_n/ρ, and the details of the energy-injection method. (5) We characterize the energy and enstrophy cascades by computing the energy and enstrophy fluxes and the mutual-friction transfer functions for all wave-number scales k.

In Chapter 6, we examine the multiscaling of structure functions in three-dimensional superfluid turbulence by using a shell-model for the three-dimensional HVBK equations. Our HVBK shell model is based on the GOY shell model. In particular, we examine the dependence of multiscaling on the normal-fluid fraction and the mutual-friction coefficients.

We hope our *in silico* studies of 2D and 3D superfluid turbulence will stimulate new experimental, numerical, and theoretical studies.